3 Lecture 2: The Standard Model

3.1 Motivation

In the previous lecture we saw that a theory for the weak interactions based on the exchange of massive vector bosons can give a good explanation to data but faces several theoretical problems, such as the lack of renormalizability and the violation of unitarity at relatively low energies.

Several authors soon speculated about the possibility that these problems could get solved by embedding the IVB hypothesis into a gauge framework. QED, the theory for the electromagnetic interactions, was a good example of such approach, as it was known to be unitary and renormalizable.

Noting the vectorial nature of both interactions, Schwinger suggested in 1957 the idea of weak and electromagnetic unification: a common theory that would describe both. Later, crucial works by Glashow [20], who correctly identified the gauge group in 1961, and by Weinberg [21] and Salam [22], who independently introduced the Higgs mechanism to account for the gauge boson masses (in 1967 and 1968, respectively), served to establish the standard theory of the electroweak interactions.

Before focusing on the electroweak part of the SM we should say a few words about the strong interactions, the other piece of the theory. The regularities found in the zoo of hadrons eventually led to the *quark model*, by Gell-Mann and Zweig, which in turn led to the idea that quarks should have an internal quantum number that allows them to respect Pauli's exclusion principle. Indeed, $J^P = \frac{3}{2}^+$ baryons such as Ω^- (sss) seemed to violate this fundamental law. Since they are formed by three quarks of the same type (or flavor) with all the spins aligned in the same direction, the spin wave function is symmetric. The ground state of these baryons has zero total angular momentum, thus also implying a symmetric spatial wave function. Consequently, the overall wave function would be totally symmetric unless quarks have an additional hidden d.o.f.. Greenberg postulated in 1964 that this additional quantum number, called color, comes in three types, hence solving the problem in the quark model. This led to the development of a gauge theory for the strong interactions (Quantum ChromoDynamics, QCD) based on the $SU(3)$ group. This theory is the second piece of the SM.

3.2 Building the SM

In this Section we will construct the SM from scratch, whereas in the next one we will discuss some of its consequences and predictions.

Steps to construct a gauge theory

Based on what we learnt in the previous lecture, one can establish general steps to construct a gauge theory. These are:

1. **Choose the gauge group.**
2. **Choose the fermion representations.**

   In the second step we must asign representations under the gauge group to the fermions. This is equivalent to defining the way in which they transform under the symmetry. In the previous examples we always considered the fundamental representation of $SU(2)$, the doublet, but other possibilities exist.

   One important check must be applied once the fermion representations are decided: the cancellation of gauge anomalies. We will come back to this issue below.
3. **Choose the scalar representations.**

   We must introduce scalar fields to break the gauge symmetry spontaneously and give masses to the massive gauge bosons.
4. **Write the most general renormalizable Lagrangian invariant under the gauge symmetry.**
5. **Minimize the scalar potential and shift the scalar fields in such a way that the minimum of the potential is located at the origin of the new scalar fields.**

Once these five steps are followed the gauge model is fully defined and one can start deriving physical consequences. Let us now go through these five steps for the SM.

Step 1: Gauge group

The first step is to choose the gauge group for our unified theory for the weak and electromagnetic interactions.

Given that the weak charged currents are of the form

$$J_\mu^{\rm cc} = \overline{\psi}_1 \gamma_\mu \left(1-\gamma_5\right) \psi_2 \equiv 2\, \overline{\psi}_{1L} \gamma_\mu \psi_{2L} \,, \tag{76}$$

with $(\psi_1, \psi_2) = (\nu_e, e^-)\,, (u,d)\,, \ldots$, the simplest possibility is to consider the $SU(2)$ group and assume that the ψ_1 and ψ_2 fermions, or more precisely, their left-handed components, form a doublet

$$\psi_L = \left(\begin{array}{c} \psi_1 \\ \psi_2 \end{array} \right)_L \,. \tag{77}$$

In this way, the $W^\pm$ gauge bosons will mediate interactions between the members of the doublet and $SU(2)$ can be denoted as $SU(2)_L$ as it only involves left-handed fermions. What about electromagnetism? The $SU(2)$ group has three generators: $T_{1,2,3}$. T_1 and T_2 combine to the $T_\pm$ generators, associated with the $W^\pm$ bosons. Could T_3 be associated with the photon? In other words: is $T_3 = Q$?

There are several arguments which show that this is not possible. Technically, one can show that Q does not close the $SU(2)$ algebra with T_+ and T_-, and thus cannot be T_3 (which necessarily does). The reason is easy: in order for Q to bea generator of $SU(2)$ the charges of a complete multiplet must add up to zero, due to the requirement that the $SU(2)$ generators must be traceless. In this case we see that the charges of ψ_1 and ψ_2 (ν_e and e^-, for instance) do not satisfy this condition. A second argument, perhaps more clear, is that while the generators $T_\pm$ will generate interactions of the V-A form, Q must generate vectorial $(\overline{\psi}_1 \gamma_\mu \psi_1)$ interactions.

This led Glashow to a simple but crucial idea: instead of just $SU(2)$, the correct gauge group for the electroweak interactions is $SU(2) \times U(1)$. This means that two gauge groups will coexist in the theory and their generators will commute. As we will see, the introduction of this extra $U(1)$ factor works.

In order to identify the nature of the additional $U(1)$ piece let us consider the first fermion family, composed by [2]

$$\nu_{eL}, e_L, e_R, u_L, u_R, d_L, d_R \,. \tag{78}$$

The electric charge operator Q is a conserved charge of the theory and can be computed using the Noether theorem from the integration of the zero component of the electromagnetic current,

$$J_\mu^{\rm em} = q_i \, \overline{\psi}_i \gamma_\mu \psi_i \quad \Rightarrow \quad Q = \int d^3x \, J_0^{\rm em} \,, \tag{79}$$

obtaining

$$\begin{aligned} Q &= \int d^3x \left(q_e e^\dagger e + q_{\nu_e} \nu_e^\dagger \nu_e + q_u u^\dagger u + q_d d^\dagger d \right) \\ &= \int d^3x \left(-e^\dagger e + \frac{2}{3} q_u u^\dagger u - \frac{1}{3} q_d d^\dagger d \right) \\ &= \int d^3x \left(-e_L^\dagger e_L - e_R^\dagger e_R + \frac{2}{3} u_L^\dagger u_L + \frac{2}{3} u_R^\dagger u_R - \frac{1}{3} d_L^\dagger d_L - \frac{1}{3} d_R^\dagger d_R \right) \,. \end{aligned} \tag{80}$$

Here we have used $q_e = -1$, $q_{\nu_e} = 0$, $q_u = \frac{2}{3}$, $q_d = -\frac{1}{3}$. Similarly, the T_3 generator is also a conserved charge of the theory and can be computed from the zero component of the weak current. Due to the $SU(2)$ underlying symmetry and the left-handed chirality of the involved fermions, the weak charged current can be generalized to

$$J_\mu^a = \overline{\psi}_L \gamma_\mu \frac{\tau_a}{2} \psi_L \,, \tag{81}$$

with $a = 1,2,3$. The currents $J_\mu^{1,2}$ can be combined to give the charged current $J_\mu^{\rm cc}$, whereas J_μ^3 is found to be

$$\begin{aligned} J_\mu^3 &= \overline{\psi}_L \gamma_\mu \frac{\tau_3}{2} \psi_L = \\ &= \frac{1}{2} \left(\begin{array}{cc} \overline{\psi}_1 & \overline{\psi}_2 \end{array} \right)_L \gamma_\mu \left(\begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array} \right) \left(\begin{array}{c} \psi_1 \\ \psi_2 \end{array} \right) \\ &= \frac{1}{2} \left(\overline{\psi}_{1L} \gamma_\mu \psi_{1L} - \overline{\psi}_{2L} \gamma_\mu \psi_{2L} \right) \,. \end{aligned} \tag{82}$$

[2] Note that we decided not to include right-handed neutrinos in this list. As we will see later in this lecture, this will have important consequences.

Group	Gauge coupling	Gauge bosons		
$SU(3)_c$	g_s	G^a_μ	$a = 1, \ldots, 8$	gluons
$SU(2)_L$	g	W^a_μ	$a = 1, \ldots, 3$	W bosons
$U(1)_Y$	g'	B_μ		B boson

Table 1: Standard Model gauge groups, couplings and bosons.

And then, the T_3 generator is found to be

$$\begin{aligned} T_3 &= \int d^3x \, J_0^3 \\ &= \frac{1}{2} \int d^3x \left({\nu_e}_L^\dagger {\nu_e}_L - e_L^\dagger e_L + u_L^\dagger u_L - d_L^\dagger d_L \right) . \end{aligned} \tag{83}$$

Now it is easy to see that the combination

$$Q - T_3 = \int d^3x \left[-\frac{1}{2} \left({\nu_e}_L^\dagger {\nu_e}_L + e_L^\dagger e_L \right) + \frac{1}{6} \left(u_L^\dagger u_L + d_L^\dagger d_L \right) - e_R^\dagger e_R + \frac{2}{3} u_R^\dagger u_R - \frac{1}{3} d_R^\dagger d_R \right] \tag{84}$$

gives the same quantum numbers to all members of an $SU(2)$ doublet. For this reason, it commutes with the $SU(2)$ generators and we can identify it with the generator of the additional $U(1)$ piece. We choose [3]

$$Y = Q - T_3 \tag{85}$$

as the generator of the $U(1)$ group and refer to Y as the "hypercharge".

We then conclude that the electroweak interactions are described by the gauge group $SU(2)_L \times U(1)_Y$. The other SM piece, the strong interactions, are described by an $SU(3)$ gauge theory associated to the three colors of quarks. Therefore,

$$SU(3)_c \times SU(2)_L \times U(1)_Y \tag{86}$$

is the complete gauge group of the SM. Table 1 summarizes this conclusion and shows how the gauge bosons and couplings are denoted.

Step 2: Fermion representations

We have already discussed fermions representations when picking up the gauge group. Left-handed fermions are doublets of $SU(2)_L$ whereas right-handed fermions are singlets (they do not transform under the gauge group and hence they do not couple to the gauge bosons). Their hypercharge is obtained from the $Y = Q - T_3$ relation, which leads to the analog of the famous Gell-Mann – Nishijima formula $Q = T_3 + Y$. Finally, quarks are in the fundamental (triplet) representation of $SU(3)_c$. All these details are summarized in Table 2, which displays the quantum numbers for the SM fermion representations. The lepton and quark doublets are denoted as

$$\ell_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad , \quad q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L . \tag{87}$$

One of the first things that one notices when looking at Table 2 is the absence of right-handed neutrinos. We then followed the original choice made by the fathers of the SM, who did not consider a ν_R representation. If introduced, the right-handed neutrino would transform as $(1,1)_0$ under the SM gauge group, where we indicate the $SU(3)_c \times SU(2)_L$ representations in brackets and the $U(1)_Y$ charge as subindex. Such a state would be a complete singlet and would not participate in gauge interactions. In what concerns the phenomenological implications of not introducing a ν_R field, there are two immediate consequences: (i) all neutrinos must be observed to have left chirality, and (ii) neutrinos must be massless (as we will see below). These two features were phenomenologically acceptable in the 60's, and thus right-handed neutrinos were not introduced for economical reasons. We will nevertheless come back to this point later.

Another important detail about Table 2 is that each fermion representation comes in three copies, known as generations or families. Even though we will generically use the notation for the first generation to refer to

[3] The normalization of the hypercharge generator is a convention and many authors prefer the definition $Y = 2\,(Q - T_3)$. One must therefore be careful when comparing different texts.

Representation	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
ℓ_L	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$
e_R	$\mathbf{1}$	$\mathbf{1}$	-1
q_L	$\mathbf{3}$	$\mathbf{2}$	$\frac{1}{6}$
u_R	$\mathbf{3}$	$\mathbf{1}$	$\frac{2}{3}$
d_R	$\mathbf{3}$	$\mathbf{1}$	$-\frac{1}{3}$

Table 2: Standard Model fermion representations. There are three generations of each representation.

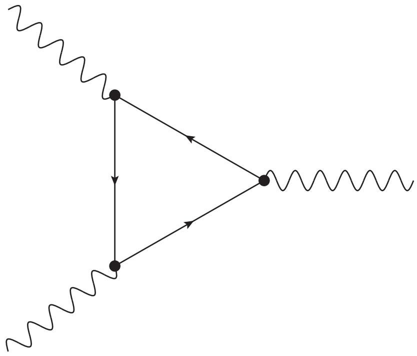


Figure 5: Generic triangle Feynman diagrams that induce unwanted gauge anomalies.

any of them, let us introduce the usual notation:

$$\textbf{1st generation:} \quad \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L , \; e_R \, , \; \begin{pmatrix} u \\ d \end{pmatrix}_L , \; u_R \, , \; d_R \tag{88}$$

$$\textbf{2nd generation:} \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L , \; \mu_R \, , \; \begin{pmatrix} c \\ s \end{pmatrix}_L , \; c_R \, , \; s_R \tag{89}$$

$$\textbf{3rd generation:} \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L , \; \tau_R \, , \; \begin{pmatrix} t \\ b \end{pmatrix}_L , \; t_R \, , \; b_R \tag{90}$$

The fact that the SM fermions are replicated in three generations does not follow from gauge invariance but it is just an experimental observation. Indeed, it would be perfectly consistent, from the theory point of view, to have only one family of fermions. We will comment on this issue in lecture 3.

We are done assigning fermion representations. At this point in the construction of a gauge theory, there is always a crucial check one must go through: one must make sure that gauge anomalies cancel.

In QFT, some loop corrections can violate a classical local conservation law derived from gauge invariance. These so-called anomalies are usually induced by Feynman diagrams such as that in Fig. 5, with fermions running in the loop and vector bosons in the external legs. Unless they cancel exactly, the presence of these diagrams can cause consistency issues that completely spoil the high-energy validity of our theory. In particular, renormalizability would not be guaranteed.

Let us consider a generic chiral theory in which left- and right-handed fermions couple differently to the gauge bosons. The interaction Lagrangian is given by

$$\mathcal{L} = -g \left(\overline{R} \gamma^\mu T_R^a R + \overline{L} \gamma^\mu T_L^a L \right) V_\mu^a \, , \tag{91}$$

where $T_{L,R}^a$ are the generators in the left and right representations of the matter fields and V_μ^a are the gauge bosons. Then the theory will be anomaly free if

$$\mathcal{A}^{abc} = \mathcal{A}_L^{abc} - \mathcal{A}_R^{abc} = 0 \, , \tag{92}$$

with

$$\mathcal{A}_{L,R}^{abc} = \mathrm{Tr} \left[\left\{ T_{L,R}^a, T_{L,R}^b \right\} T_{L,R}^c \right] \, . \tag{93}$$

Representation	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
Φ	**1**	**2**	$\frac{1}{2}$

Table 3: Standard Model scalar representation.

We then see that anomalies are likely to appear in model with $T_L^a \neq T_R^a$. For this reason, in case of the SM we must be concerned about the electroweak group $SU(2)_L \times U(1)_Y$. In fact, it is possible to show that the only relevant triangles are $SU(2)_L^2\, U(1)_Y$ and $U(1)_Y^3$, given by

$$SU(2)_L^2\, U(1)_Y: \quad \mathrm{Tr}\left[\left\{\tau^a, \tau^b\right\} Y\right] = \mathrm{Tr}\left[\left\{\tau^a, \tau^b\right\}\right]\, \mathrm{Tr}\left[Y\right] \propto \sum_{\text{doublets}} Y\,, \tag{94}$$

$$U(1)_Y^3: \quad \mathrm{Tr}\left[Y^3\right] \propto \sum_{\text{fermions}} Y^3\,, \tag{95}$$

and then the computation of the relevant $\mathcal{A}_{LLY}^{abc}$ and $\mathcal{A}_{YYY}^{abc}$ anomalies just requires evaluating these two sums. Notice that for the first one we just have to evaluate the sum on the $SU(2)_L$ doublets, since for singlets there is no contribution to the anomaly as they do not couple to the $SU(2)_L$ gauge bosons. For the second, in contrast, one must compute both sums (left- and right-handed fermions) and substract their contributions. One finds

$$\mathcal{A}_{LLY}^{abc} \propto \sum_{\text{doublets}} Y = Y(\ell_L) + 3\, Y(q_L) = -\frac{1}{2} + 3 \cdot \frac{1}{6} = 0\,, \tag{96}$$

and

$$\mathcal{A}_{YYY}^{abc} \propto \sum_{\text{fermions}} Y_L^3 - Y_R^3 = \left[2 \cdot \left(-\frac{1}{2}\right)^3 + 2 \cdot 3 \cdot \left(\frac{1}{6}\right)^3\right] - \left[(-1)^3 + 3 \cdot \left(\frac{2}{3}\right)^3 + 3 \cdot \left(-\frac{1}{3}\right)^3\right] = 0\,. \tag{97}$$

The factors of 3 in these two evaluations are due to the 3 quark colors and the factors of 2 come from the fact that left-handed fermions are doublets (and thus they have multiplicity 2).

Therefore, we conclude that the SM is anomaly free. This is true for each complete generation of fermions (a fact that was used, for example, to predict the existence of the top quark) due to a *conspiracy* between the quark and lepton sectors, which cancel each other's anomaly perfectly.

Step 3: Scalar representations

In order to break the gauge symmetry one must introduce scalar representations. In the SM one takes the simplest possibility: a single scalar doublet Φ,

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}\,, \tag{98}$$

with $Y = 1/2$ and singlet under $SU(3)_c$, as summarized in Table 3. This doublet is usually called *the Higgs doublet.*

Step 4: Most general Lagrangian

Since we are mostly interested in the electroweak sector we will omit $SU(3)_c$ interactions from now on.

With the ingredients introduced so far, the most general Lagrangian invariant under $SU(3)_c \times SU(2)_L \times U(1)_Y$ is

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{kin}} + \mathcal{L}_{\Phi} - \mathcal{L}_Y\,. \tag{99}$$

The first piece is the pure gauge Lagrangian,

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} W_{\mu\nu}^a W_a^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}\,, \tag{100}$$

with the gauge-field tensors

$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g \epsilon^{abc} W_\mu^b W_\nu^c\,, \tag{101}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu\,. \tag{102}$$

The second piece, $\mathcal{L}_{\rm kin}$, corresponds to the fermion kinetic terms in which the usual derivatives have been replaced by covariant derivatives,

$$\mathcal{L}_{\rm kin} = \sum_\psi \overline{\psi}\, i\gamma^\mu D_\mu \psi \,, \tag{103}$$

with $\psi = \{\ell_L, e_R, q_L, u_R, d_R\}$. The covariant derivative can generally be written as

$$D_\mu \psi = \left(\partial_\mu - ig\vec{T}\vec{W}_\mu - ig' Y B_\mu\right)\psi \,. \tag{104}$$

For instance, for the lepton doublet ℓ_L this is

$$D_\mu \ell_L = \left(\partial_\mu - ig\frac{\vec{\tau}}{2}\vec{W}_\mu + i\frac{g'}{2}B_\mu\right)\ell_L \,, \tag{105}$$

whereas for the lepton singlet one has

$$D_\mu e_R = \left(\partial_\mu + ig' B_\mu\right) e_R \,. \tag{106}$$

$\mathcal{L}_\Phi$ includes the kinetic term for the Φ scalar doublet (with the usual derivatives replaced by covariant ones) and its scalar potential,

$$\mathcal{L}_\Phi = \left(D_\mu \Phi\right)^\dagger D^\mu \Phi - V(\Phi) \,, \tag{107}$$

with

$$D_\mu \Phi = \left(\partial_\mu - ig\frac{\vec{\tau}}{2}\vec{W}_\mu - i\frac{g'}{2}B_\mu\right)\Phi \tag{108}$$

and

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda \left(\Phi^\dagger \Phi\right)^2 \,. \tag{109}$$

We note that $V(\Phi)$ is the most general scalar potential allowed by $SU(2)_L \times U(1)_Y$. For example, the gauge symmetry forbids a possible Φ^3 term. Finally, $\mathcal{L}_Y$ contains Yukawa interactions allowed by the gauge symmetry,

$$\mathcal{L}_Y = Y_e\, \overline{\ell}_L \Phi e_R + Y_u\, \overline{q}_L \widetilde{\Phi} u_R + Y_d\, \overline{q}_L \Phi d_R + \text{h.c.} \,, \tag{110}$$

where $\widetilde{\Phi} = i\tau_2 \Phi^*$ is the conjugate of Φ with well defined transformations (doublet of $SU(2)_L$ with $Y = -1/2$). We note that $Y_{e,u,d}$ are generic 3×3 complex matrices since all fermions in this Lagrangian come in three families.

We emphasize once again that this Lagrangian does not contain any mass term for the fermions and gauge bosons. These are all forbidden by the gauge symmetry.

Step 5: Symmetry breaking

As we already know, the quartic coupling λ must be positive for the potential to be bounded from below. Now, if $\mu^2 < 0$ the minimum of the potential is not at $\langle \Phi \rangle = 0$, but at

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad , \quad \text{with}\, v = \sqrt{\frac{-\mu^2}{\lambda}} \,. \tag{111}$$

This, known as the Higgs VEV, spontaneously breaks the gauge symmetry. But what is the remnant symmetry (if any) after symmetry breaking? It is easy to see that the four generators of $SU(2)_L \times U(1)_Y$ are broken in the vacuum given by $\langle \Phi \rangle$. The vacuum is left invariant by a generator G if

$$e^{i\alpha G} \langle \Phi \rangle = \langle \Phi \rangle \,, \tag{112}$$

which, for an infinitesimal transformation ($\alpha \ll 1$), leads to

$$e^{i\alpha G} \langle \Phi \rangle \simeq \left(1 + i\alpha G\right) \langle \Phi \rangle \tag{113}$$

which implies $G\langle\Phi\rangle = 0$. In this case we say that "G annihilates the vacuum". We can now apply all four generators of the electroweak gauge group to the vacuum. We find

$$T_1\langle\Phi\rangle = \frac{\tau_1}{2}\langle\Phi\rangle = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} \frac{v}{\sqrt{2}} \\ 0 \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \text{Broken} \tag{114}$$

$$T_2\langle\Phi\rangle = \frac{\tau_2}{2}\langle\Phi\rangle = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} = -\frac{i}{2}\begin{pmatrix} \frac{v}{\sqrt{2}} \\ 0 \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \text{Broken} \tag{115}$$

$$T_3\langle\Phi\rangle = \frac{\tau_3}{2}\langle\Phi\rangle = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} = -\frac{1}{2}\begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \text{Broken} \tag{116}$$

$$Y\langle\Phi\rangle = Y_\Phi\langle\Phi\rangle = +\frac{1}{2}\langle\Phi\rangle = \frac{1}{2}\begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \text{Broken} \tag{117}$$

However, if we examine the effect of the electric charge operator Q on the vacuum we find

$$Q\langle\Phi\rangle = (T_3 + Y)\langle\Phi\rangle = \left(\frac{\tau_3}{2} + Y_\Phi\right)\langle\Phi\rangle = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \text{Unbroken} \tag{118}$$

We find that electric charge is unbroken even after SSB. Therefore, the resultant symmetry breaking pattern is

$$SU(2)_L \times U(1)_Y \to U(1)_{\text{em}} \tag{119}$$

and the Higgs VEV preserves electric charge conservation.

With these five steps the SM is fully defined. In the next section we will derive some consequences and predictions of the model.

3.3 Consequences and predictions

Gauge boson masses

In order to compute the particle spectra we start by going to the unitary gauge, in which the would-be Goldstones do not appear and the interpretation of the analytical results is more transparent. Analogously to what we saw in the first lecture, in this gauge Φ is given by

$$\Phi = \frac{1}{\sqrt{2}}\,(v+h)\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\,(v+h)\,\chi\,, \tag{120}$$

where $h = h(x)$ is the physical scalar field with vanishing VEV ($\langle h\rangle$). The gauge boson masses are contained in the $(D_\mu\Phi)^\dagger D^\mu\Phi$ piece of $\mathcal{L}_\Phi$. Since we are not interested at the moment in the interactions, we can concentrate on the terms

$$\mathcal{L}_m^{\text{GB}} = \Phi^\dagger\left(ig\frac{\vec{\tau}}{2}\vec{W}_\mu + i\frac{g'}{2}B_\mu\right)\left(-ig\frac{\vec{\tau}}{2}\vec{W}^\mu - i\frac{g'}{2}B^\mu\right)\Phi = \frac{v^2}{8}\chi^T M_\mu M^\mu\chi\,, \tag{121}$$

with

$$\begin{aligned} M_\mu =& g\vec{\tau}\vec{W}_\mu + g'B_\mu \\ =& \begin{pmatrix} gW_\mu^3 + g'B_\mu & g\left(W_\mu^1 - iW_\mu^2\right) \\ g\left(W_\mu^1 + iW_\mu^2\right) & -gW_\mu^3 + g'B_\mu \end{pmatrix} \\ =& \begin{pmatrix} gW_\mu^3 + g'B_\mu & \sqrt{2}gW_\mu^+ \\ \sqrt{2}gW_\mu^- & -gW_\mu^3 + g'B_\mu \end{pmatrix}, \end{aligned} \tag{122}$$

where we have identified the charged mass eigenstates

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp iW_\mu^2\right). \tag{123}$$

Now, operating we get

$$\mathcal{L}_m^{\rm GB} = \frac{v^2}{8}\left[2g^2 W_\mu^- W^{+\mu} + \left(g' B_\mu - g W_\mu^3\right)^2\right] \equiv m_W^2 W_\mu^- W^{+\mu} + \frac{1}{2}\left(V_\mu^0\right)^T \mathcal{M}_{V^0}^2 V^{0\mu} \, . \tag{124}$$

The $W_\mu^{1,2}$ gauge bosons have been combined into a pair of charged gauge bosons $W_\mu^\pm$, with mass

$$m_W^2 = \frac{g^2 v^2}{4} \, . \tag{125}$$

On the other hand, the neutral gauge bosons $V_\mu^0 = \left(B_\mu, W_\mu^3\right)^T$ mix, with mass matrix

$$\mathcal{M}_{V^0}^2 = \frac{v^2}{4}\left(\begin{array}{cc} g'^2 & -gg' \\ -gg' & g^2 \end{array}\right) \, . \tag{126}$$

We must diagonalize this matrix to get the mass eigenstates and eigenvalues. This is done by means of the following unitary transformation

$$V_\mu^0 = \left(\begin{array}{c} B_\mu \\ W_\mu^3 \end{array}\right) = \left(\begin{array}{cc} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{array}\right)\left(\begin{array}{c} A_\mu \\ Z_\mu \end{array}\right) \equiv R_{V^0}\widehat{V}_\mu^0 \, , \tag{127}$$

which is equivalent to the linear combinations

$$A_\mu = \cos\theta_W \, B_\mu + \sin\theta_W \, W_\mu^3 \, , \tag{128}$$

$$Z_\mu = -\sin\theta_W \, B_\mu + \cos\theta_W \, W_\mu^3 \, . \tag{129}$$

The unitary matrix R_{V^0} diagonalizes $\mathcal{M}_{V^0}^2$ as

$$\begin{aligned} \left(V_\mu^0\right)^T \mathcal{M}_{V^0}^2 V^{0\mu} &= \left(\widehat{V}_\mu^0\right)^T R_{V^0}^T \mathcal{M}_{V^0}^2 R_{V^0} \widehat{V}^{0\mu} \equiv \left(\widehat{V}_\mu^0\right)^T \widehat{\mathcal{M}}_{V^0}^2 \widehat{V}^{0\mu} \\ &= \left(\begin{array}{cc} A_\mu & Z_\mu \end{array}\right)\left(\begin{array}{cc} m_A^2 & 0 \\ 0 & m_Z^2 \end{array}\right)\left(\begin{array}{c} A_\mu \\ Z_\mu \end{array}\right) \, , \end{aligned} \tag{130}$$

with

$$m_A^2 = 0 \, , \tag{131}$$

$$m_Z^2 = \frac{v^2}{4}\left(g^2 + g'^2\right) \, , \tag{132}$$

and

$$\sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}} \quad , \quad \cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}} \, , \tag{133}$$

or, equivalently, $\tan\theta_W = \frac{g'}{g}$. The angle of rotation θ_W is usually referred to as the "weak mixing angle" [4].

As Eq. (131) clearly shows, the A_μ gauge boson remains massless after SSB. This gauge boson can thus be identified with the photon, which must be massless due to the conservation of $U(1)_{\rm em}$. The other neutral gauge boson, the Z-boson, is massive. We find an important relation between its mass and that of the W-boson, given in Eq. (125),

$$\rho = \frac{m_W^2}{\cos^2\theta_W \, m_Z^2} = 1 \, . \tag{134}$$

This ratio, which also represents the relative strength of the neutral and charged interactions (as we will see below) is equal to 1 only due to the specific scalar sector that we have chosen. If instead of just the doublet Φ we had introduced other scalar representations with non-zero VEVs, the ρ parameter could have easily departed from 1. Therefore, $\rho = 1$ is a definite (tree-level) prediction of the SM.

[4] It is also common to use the name "Weinberg angle", but this seems to be unfair as the parameter appeared for the first time in Glashow's classical paper [20].

Scalar mass: the Higgs boson

After SSB the model contains the real scalar field h. This field is the so-called Higgs boson and can be seen as the footprint left by SSB (the Higgs mechanism). Its mass can be easily derived by replacing the shifted parameterization of Φ into $V(\Phi)$. One finds

$$V(\Phi) \supset -\mu^2 h^2 = \frac{1}{2} m_h^2 h^2 \,, \tag{135}$$

which then implies

$$m_h = \sqrt{-2\mu^2} \,, \tag{136}$$

with $m_h^2 > 0$, since $\mu^2 < 0$.

Fermion masses

We now turn our attention to the fermion masses. As already pointed out, all fermions are massless before SSB since their mass terms are forbidden by the gauge symmetry. However, their masses are generated after SSB thanks to the Yukawa terms in $\mathcal{L}_Y$. For example, for the leptons this is given by [5]

$$\mathcal{L}_Y^\ell = Y_e \bar{\ell}_L \Phi e_R + \text{h.c.} \,. \tag{137}$$

Then, using the already familiar expression for Φ in the unitary gauge, one finds

$$\begin{aligned} \mathcal{L}_Y^\ell =& Y_e \bar{\ell}_L \Phi e_R + \text{h.c.} = \frac{1}{\sqrt{2}} (v+h) Y_e \begin{pmatrix} \bar{\nu} & \bar{e} \end{pmatrix}_L \begin{pmatrix} 0 \\ 1 \end{pmatrix} e_R + \text{h.c.} \\ =& \mathcal{M}_e \bar{e}_L e_R + \frac{\mathcal{M}_e}{v} h \bar{e}_L e_R + \text{h.c.} \,, \end{aligned} \tag{138}$$

where

$$\mathcal{M}_e = \frac{v}{\sqrt{2}} Y_e \tag{139}$$

is the 3×3 mass matrix for the charged leptons. We note that neutrinos do not get masses this way. This fact can be traced back to the absence of right-handed neutrinos in the theory.

Similarly, one gets 3×3 mass matrices for the up- and down-type quarks. The complete fermion mass Lagrangian is

$$\mathcal{L}_m^F = \mathcal{M}_e \bar{e}_L e_R + \mathcal{M}_u \bar{u}_L u_R + \mathcal{M}_d \bar{d}_L d_R + \text{h.c.} \,, \tag{140}$$

with

$$\mathcal{M}_f = \frac{v}{\sqrt{2}} Y_f \,, \tag{141}$$

with $f = e, u, d$. It is definitely remarkable that the same mechanism that gives mass to the gauge bosons (SSB), also gives a mass to the fermions. Now, in general these three mass matrices are not diagonal, since the $Y_{e,u,d}$ are general complex matrices. In order to obtain mass eigenstates and eigenvalues, the mass matrices in Eq. (141) must be brought to a diagonal form. Since all mass in terms in $\mathcal{L}_m^F$ are of Dirac type, this must be done by means of biunitary transformations.

Given a matrix $\mathcal{M}$, there exist two unitary matrices U and V ($UU^\dagger = U^\dagger U = \mathbb{I}$ and $VV^\dagger = V^\dagger V = \mathbb{I}$) such that

$$U^\dagger \mathcal{M} V = \widehat{\mathcal{M}} \,, \tag{142}$$

where $\widehat{\mathcal{M}}$ is diagonal with positive eigenvalues. U and V can be found by noticing that they diagonalize $\mathcal{M}\mathcal{M}^\dagger$ and $\mathcal{M}^\dagger \mathcal{M}$, respectively:

$$U^\dagger \mathcal{M} V = \widehat{\mathcal{M}} \quad \Rightarrow \quad \widehat{\mathcal{M}}^2 = U^\dagger \mathcal{M} V V^\dagger \mathcal{M}^\dagger U = U^\dagger \mathcal{M}\mathcal{M}^\dagger U \,, \tag{143}$$

$$U^\dagger \mathcal{M} V = \widehat{\mathcal{M}} \quad \Rightarrow \quad \widehat{\mathcal{M}}^2 = V^\dagger \mathcal{M}^\dagger U U^\dagger \mathcal{M} V = V^\dagger \mathcal{M}^\dagger \mathcal{M} V \,. \tag{144}$$

In our case, this can be applied to $\mathcal{M}_{e,u,d}$. The unitary matrices U and V are independent transformations of the left- and right-handed fermions, respectively, connecting the original gauge bases to the mass bases ($\widehat{e}$, $\widehat{u}$, $\widehat{d}$),

$$f_L = U_f \widehat{f}_L \,, \tag{145}$$

$$f_R = V_f \widehat{f}_R \,, \tag{146}$$

[5] We have defined the Yukawa Lagrangian $\mathcal{L}_Y$ with a convenient negative sign in Eq. (99) so that the resulting mass terms will be proportional to Y_f, and not to $-Y_f$. This is due to the fact that mass terms come with a negative sign in the Lagrangian, see for instance the Dirac Lagrangian in Eq. (25).

with $f = e, u, d$, and then

$$\widehat{\mathcal{M}}_f = U_f^\dagger \mathcal{M}_f V_f \tag{147}$$

is the diagonal mass matrix in the mass bases. For instance, in case of the charged leptons one finds

$$\widehat{\mathcal{M}}_e = U_e^\dagger \mathcal{M}_e V_e = \text{diag}\left(m_e, m_\mu, m_\tau\right) \,, \tag{148}$$

with $m_{e,\mu,\tau}$ the physical masses of the SM charged leptons.

The charged current

In order to obtain the interaction Lagrangian for the W-boson and identify the currents with those of the V-A theory we must have a look at the fermion gauge interactions in $\mathcal{L}_{\text{kin}}$. The relevant terms are

$$\begin{aligned}\mathcal{L}_{\text{kin}} \supset & \, \overline{\ell}_L \left(g \frac{\vec{\tau}}{2} \vec{W}_\mu - \frac{g'}{2} B_\mu \right) \gamma^\mu \ell_L + \overline{q}_L \left(g \frac{\vec{\tau}}{2} \vec{W}_\mu + \frac{g'}{6} B_\mu \right) \gamma^\mu q_L \\ & - \overline{e}_R g' B_\mu \gamma^\mu e_R + \overline{u}_R \, \frac{2}{3} \, g' B_\mu \gamma^\mu u_R - \overline{d}_R \, \frac{1}{3} \, g' B_\mu \gamma^\mu d_R \\ = & \, g J_\mu^1 W^{1\mu} + g J_\mu^2 W^{2\mu} + g J_\mu^3 W^{3\mu} + g' J_\mu^Y B^\mu \,,\end{aligned} \tag{149}$$

where $J_\mu^{1,2,3,Y}$ are implicitly defined in the previous expression,

$$J_\mu^1 = \frac{1}{2} \left(\overline{\nu}_L \gamma_\mu e_L + \overline{u}_L \gamma_\mu d_L + \text{h.c.} \right) \,, \tag{150}$$

$$J_\mu^2 = - \frac{i}{2} \left(\overline{\nu}_L \gamma_\mu e_L + \overline{u}_L \gamma_\mu d_L - \text{h.c.} \right) \,, \tag{151}$$

$$J_\mu^3 = \frac{1}{2} \left(\overline{\nu}_L \gamma_\mu \nu_L - \overline{e}_L \gamma_\mu e_L + \overline{u}_L \gamma_\mu u_L - \overline{d}_L \gamma_\mu d_L \right) \,, \tag{152}$$

$$J_\mu^Y = \frac{1}{2} \left(-3 \, \overline{\nu}_L \gamma_\mu \nu_L - 3 \, \overline{e}_L \gamma_\mu e_L + \overline{u}_L \gamma_\mu u_L + \overline{d}_L \gamma_\mu d_L - 6 \, \overline{e}_R \gamma_\mu e_R + 4 \, \overline{u}_R \gamma_\mu u_R - 2 \, \overline{d}_R \gamma_\mu d_R \right) \,. \tag{153}$$

The first two currents (J_μ^1 and J_μ^2) are charged (since $W_\mu^{1,2}$ combine to give $W_\mu^\pm$) and the last two are neutral (since W_μ^3 and B_μ lead to A_μ and Z_μ). Let us first focus on the charged current. This is given by

$$\begin{aligned}\mathcal{L}_{\text{cc}} = & g J_\mu^1 W^{1\mu} + g J_\mu^2 W^{2\mu} \\ = & \frac{g}{\sqrt{2}} \left[\left(J^1 + i J^2 \right)_\mu W^{+\mu} + \left(J^1 - i J^2 \right)_\mu W^{-\mu} \right] \\ = & \frac{g}{\sqrt{2}} \left[J_\mu^+ W^{+\mu} + \text{h.c.} \right] \,,\end{aligned} \tag{154}$$

where we have used the definition of the $W_\mu^\pm$ bosons, which can be inversed to give

$$W_\mu^1 = \frac{1}{\sqrt{2}} \left(W^+ + W^- \right)_\mu \,, \tag{155}$$

$$W_\mu^2 = \frac{i}{\sqrt{2}} \left(W^+ - W^- \right)_\mu \,, \tag{156}$$

and have defined $J_\mu^+ = J_\mu^1 + i J_\mu^2$. Using now our expressions for J_μ^1 and J_μ^2, Eqs. (150) and (151), we finally get

$$J_\mu^+ = \overline{\nu}_L \gamma_\mu e_L + \overline{u}_L \gamma_\mu d_L = \frac{1}{2} \left[\overline{\nu} \gamma_\mu (1 - \gamma_5) e + \overline{u} \gamma_\mu (1 - \gamma_5) d \right] = \frac{1}{2} J_\mu \,, \tag{157}$$

where J_μ is the V-A current we introduced in the V-A and IVB theories. Therefore, we can do the same identification with the low-energy effective theory, leading to

$$\frac{g^2}{8 m_W^2} = \frac{G_F}{\sqrt{2}} \,. \tag{158}$$

Using now the W-boson mass in Eq.(125), one finds

$$v = \left(\sqrt{2} G_F \right)^{-1/2} \simeq 246 \, \text{GeV} \,, \tag{159}$$

where we have used the numerical value $G_F = 1.166 \cdot 10^{-5} \, \text{GeV}^{-2}$. The message behind Eq. (159) is clear: the electroweak VEV v and the Fermi constant are actually the same quantity. Even more: the Fermi scale is generated by the Higgs doublet VEV!

There is one more thing that we must do to get $\mathcal{L}_{\rm cc}$ in terms of physical states (mass eigenstates). As explained above, we must rotate to the fermion mass bases using the U_f, V_f unitary transformations. Therefore, the quark charged current interaction Lagrangian becomes

$$\mathcal{L}_{\rm cc}^q = \frac{g}{\sqrt{2}} \overline{\widehat{u}}_L \gamma_\mu V_{\rm CKM} \widehat{d}_L W^{+\mu} + \text{h.c.} \,, \tag{160}$$

where we have defined

$$V_{\rm CKM} = U_u^\dagger U_d \,, \tag{161}$$

a 3×3 unitary matrix, obtained from the product of the left u and d rotations. This is the famous Cabibbo-Kobayashi-Maskawa (CKM) matrix, introduced by Kobayashi and Maskawa in 1973 [23]. The elements of this matrix determine the relative size of different quark flavor transitions in charged current interactions.

It is instructive to count the number of physical parameters in $V_{\rm CKM}$. A general $n \times n$ unitary matrix has

$$n^2 \,\text{real parameters :} \quad \frac{n(n-1)}{2} \,\text{angles} \quad + \quad \frac{n(n+1)}{2} \,\text{phases} \,. \tag{162}$$

However, not all these parameters are physical, since one can absorbe some phases by rephasing the fields

$$u_i \to e^{i\phi_i} u_i \quad , \quad d_j \to e^{i\theta_j} d_j \quad \Rightarrow \quad V_{\rm CKM}^{ij} \to V_{\rm CKM}^{ij} \, e^{i(\theta_j - \phi_i)} \,, \tag{163}$$

and in this way one can eliminate $2n - 1$ unphysical phases. Therefore, we are left with

$$(n-1)^2 \,\text{physical real parameters :} \quad \frac{n(n-1)}{2} \,\text{angles} \quad + \quad \frac{(n-1)(n-2)}{2} \,\text{phases} \,. \tag{164}$$

Let us now consider two cases:

- **2 generations ($n = 2$):** 1 angle

If we only had two quark generations, $\widehat{u}_i = (\widehat{u}, \widehat{c})$ and $\widehat{d}_i = (\widehat{d}, \widehat{s})$, the CKM matrix would be parameterized by a single angle,

$$V_{\rm CKM}^{2\times 2} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \,, \tag{165}$$

where θ_c is the Cabibbo angle [12] discussed in the first lecture.

- **3 generations ($n = 3$):** 3 angles + 1 phase

For the realistic case of 3 quark generations, the CKM matrix is parameterized in terms of 3 angles and 1 imaginary phase,

$$V_{\rm CKM}^{3\times 3} = \begin{pmatrix} c_{13} c_{12} & s_{12} c_{13} & s_{13} e^{-i\delta} \\ -s_{12} c_{23} - s_{23} s_{13} c_{12} e^{i\delta} & c_{23} c_{12} - s_{23} s_{13} s_{12} e^{i\delta} & s_{23} c_{13} \\ s_{23} s_{12} - s_{13} c_{23} c_{12} e^{i\delta} & -s_{23} c_{12} - s_{13} s_{12} c_{23} e^{i\delta} & c_{23} c_{13} \end{pmatrix} \,, \tag{166}$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. The fact that a phase appears in the quark mixing matrix is crucial to allow for CP violating effects in the quark sector. As we have seen, this requires at least 3 quark generations.

Let us now consider the charged current in the lepton sector. We could in principle proceed in the same way, rotating e_L and ν_L to their mass eigenstate by means of unitary transformations. However, neutrinos are massless, an thus completely degenerate. This implies that $\nu_L = \widehat{\nu}_L$ and rotations in the neutrino *flavor space* have no effect (they leave the physics unchanged). Therefore, one can always select a specific neutrino rotation matrix U_ν such that the resulting $\mathcal{L}_{\rm cc}^\ell$ is as simple as possible. In particular, one can choose $U_\nu = U_e$, so that $\mathcal{L}_{\rm cc}^\ell$ includes the product $U_e^\dagger U_e = \mathbb{I}$, which leads to simply

$$\mathcal{L}_{\rm cc}^\ell = \frac{g}{\sqrt{2}} \overline{\widehat{\nu}}_L \gamma_\mu \widehat{e}_L W^{+\mu} + \text{h.c.} \,. \tag{167}$$

The absence of a leptonic mixing matrix, analog of the CKM matrix in the quark sector, is hence a consequence of the neutrinos being massless.

The neutral currents

Let us now discuss the neutral currents. These are given by

$$\begin{aligned}\mathcal{L}_{\rm nc} &= g J^3_\mu W^{3\mu} + g' J^Y_\mu B^\mu \\ &= g J^3_\mu \left(\cos\theta_W Z^\mu + \sin\theta_W A^\mu\right) + g' J^Y_\mu \left(-\sin\theta_W Z^\mu + \cos\theta_W A^\mu\right) \\ &= \left(g \sin\theta_W J^3_\mu + g' \cos\theta_W J^Y_\mu\right) A^\mu + \left(g\cos\theta_W J^3_\mu - g' \sin\theta_W J^Y_\mu\right) Z^\mu \, . \end{aligned} \tag{168}$$

One can now make use of the definition of θ_W to show that $g \sin\theta_W = g' \cos\theta_W$ and identify this combination as e, the fundamental electric charge,

$$e = g \sin\theta_W = g' \cos\theta_W \, . \tag{169}$$

Moreover, one can also check by direct computation that

$$J^{\rm em}_\mu = J^3_\mu + J^Y_\mu = \sum_f q_f \overline{f} \gamma_\mu f \, , \tag{170}$$

as expected due to the generators relation $Q = T_3 + Y$. With these two inputs we can rewrite

$$\begin{aligned}\mathcal{L}_{\rm nc} &= e J^{\rm em}_\mu A^\mu + \left[g\cos\theta_W J^3_\mu - g'\sin\theta_W \left(J^{\rm em}_\mu - J^3_\mu\right)\right] Z^\mu \\ &= e J^{\rm em}_\mu A^\mu + \frac{g}{\cos\theta_W} \left(J^3_\mu - \sin^2\theta_W J^{\rm em}_\mu\right) Z^\mu \, , \end{aligned} \tag{171}$$

where we just used $g' = g \tan\theta_W$ and basic trigonometry. We have then recovered the QED Lagrangian, with A^μ the photon as anticipated, and new neutral currents mediated by the massive Z-boson.

Before concluding our discussion of the neutral current we should make an observation. As for the charged currents, we should now rotate the fermion gauge eigenstates into the physical mass eigenstates. However, just by looking at the form of J^3_μ and J^Y_μ (or $J^{\rm em}_\mu$), we see that all neutral currents are of the form

$$\overline{f}_X \gamma_\mu f_X \, , \tag{172}$$

with $X = L, R$. Therefore, when we transform $f \to \widehat{f}$, the rotation matrices cancel out since they are unitary: $U^\dagger U = V^\dagger V = \mathbb{I}_{3\times 3}$. This implies that we can simply replace the gauge eigenstates by the mass eigenstates in Eq. (171) without introducing any rotation matrix in the neutral currents. Furthermore, this in turn means that, in contrast to the charged currents where the off-diagonal terms of $V_{\rm CKM}$ induce flavor violating transitions (such as $W^+ \to \overline{s} u$), neutral currents conserve flavor and processes like $Z \to \overline{u} c$ cannot take place at tree-level.

The absence of flavor changing neutral currents (FCNC) at tree-level is caused by the fact that fermion families are exact replicas: fermions with the same charge and chirality have the same gauge quantum numbers. This was the original motivation that led Glashow, Iliopoulos and Maiani (GIM) [24] to postulate the existence of the charm quark, with the same quantum numbers as the up quark. As we see, the GIM mechanism, as we currently know the absence of tree-level FCNCs due to family replication, is perfectly understood in the framework of the SM.

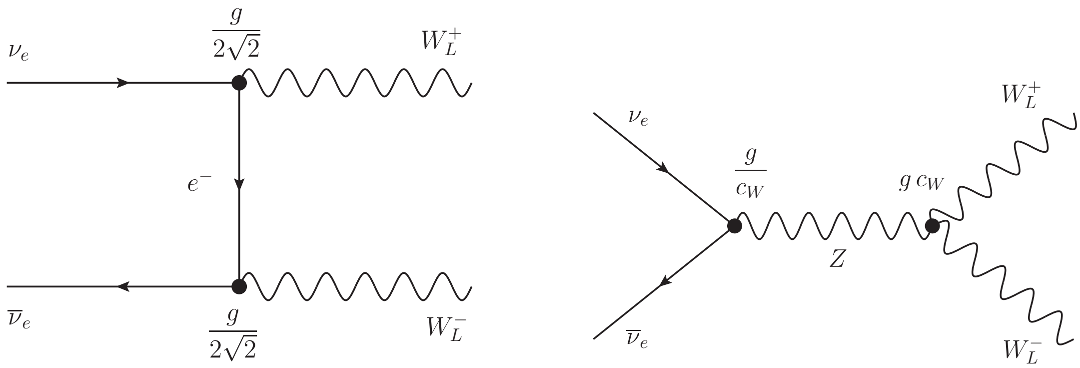
Unitarity and renormalizability

Does the SM solve the problems of the IVB: unitarity and renormalizability? Indeed it does!

For instance, let us consider the scattering process $\nu_e \bar{\nu}_e \to W^+_L W^-_L$, with $W^\pm_L$ longitudinally polarized W-bosons. We saw in lecture 1 that the IVB leads to unitarity violation in this process since $\sigma \propto s$ grows with the energy. In the SM, however, this is not the case. Now, this scattering receives two contributions, from the diagrams shown in Fig. 6. It is possible to show that th dangerous terms leading to the growth of σ with the energy are present in both contributions, but they come with opposite signs and cancel exactly in the total amplitude. The reason behind this cancellation is the gauge symmetry. Other examples of this good high-energy behavior exist. A famous example is $e^+ e^- \to W^+_L W^-_L$. Again, the cancellation requires to include all contributions, including in this case the s-channel exchange of a Higgs boson.

The proof of the renormalizability of the SM was given by Veltman and 't Hooft [25–27] in a series of works in 1971 and 1972. In fact, the proof extends to all gauge theories, with or without SSB.

Therefore, with unitarity and renormalizability saved, these two consistency issues in the pre-SM theories are no longer a problem. Finally, a consistent theory for the electromagnetic and weak interactions has been built.



(a) Also in IVB theory (b) New in the SM

Figure 6: Feynman diagrams leading to $\nu_e \bar{\nu}_e \to W_L^+ W_L^-$ in the SM. The diagram on the left was also present in the IVB theory, whereas the one on the right is a new contribution in the SM.

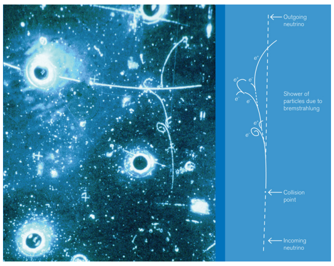


Figure 7: A picture, taken and scanned in 1972, showing a neutral current interaction taking place at the Gargamelle bubble chamber. The neutrino, which leaves no track in the detector, entered the bubble chamber from the bottom of this image and hit an electron by exchanging a neutral Z-boson.

The discovery of the W and Z bosons

After such a long theoretical discussion it is time to focus again on experimental facts. Having an elegant theory does not guarantee that it describes reality.

By the beginning of the 70's, a viable theory for the electromagnetic and weak interactions was proposed. This was a clear challenge for the experimental groups, which had to show whether or not this theory was connected to the real world. The first great discovery was that of neutral currents by the Gargamelle bubble chamber at CERN in 1973. This huge detector photographed the tracks of a few electrons suddenly starting to move, seemingly of their own accord (see Fig. 7). This was interpreted as a neutrino interacting with the electron by the exchange of an unseen Z-boson.

Although this already allowed to get some information about the underlying the theory (the increasingly popular SM), the next required step was the actual discovery of the intermediate bosons exchanged in the electroweak interactions: the W and Z bosons. This came a few years later, in 1983, with observations in the UA1 and UA2 experiments at the CERN Super Proton Synchrotron (SPS), a collider with a high enough center of mass energy. This strong *confirmation* gave the final and decisive support to the electroweak theory.

SM parameters

Before concluding, let us consider the free parameters in the SM and give, for future reference, their measured values. The free fundamental Lagrangian parameters in the electroweak sector of the SM are:

- **Fundamental parameters:** g, g', v, λ, Y_f

In practice, these fundamental parameters are traded for other derived parameters, more directly connected to experimental measurements:

- **Derived parameters:** α, m_W, m_Z, m_h, m_f, V_{CKM}

Here m_f are the masses of the SM fermions and α is the electromagnetic fine structure constant. Although these expressions have been given already, let us rewrite the connection between the derived parameters and the fundamental ones. These is obtained via the relations

$$g = \frac{e}{\sin\theta_W} \quad , \quad g' = \frac{e}{\cos\theta_W} \, , \tag{173}$$

Parameter	Meaning	Experimental value	Measured by
α^{-1}	Fine structure constant (inverse)	137.035999074(44)	Harvard cyclotron (g_e)
m_W	W-boson mass	(80.387 ± 0.016) GeV	LEP2 / Tevatron / LHC
m_Z	Z-boson mass	(91.1876 ± 0.021) GeV	LEP1 / SLD
m_h	Higgs boson mass	(125.6 ± 0.4) GeV	LHC

Table 4: Standard Model free parameters as quoted by the Particle Data Group (PDG) [28]. The fine structure constant α is given at $q^2 = 0$.

as well as

$$\alpha = \frac{e^2}{4\pi} \quad , \quad m_W = \frac{gv}{2} \quad , \quad m_Z = \frac{m_W}{\cos\theta_W} \quad , \quad m_h = \sqrt{2\lambda}\, v \quad , \quad m_f = \frac{v}{\sqrt{2}} Y_f \, . \tag{174}$$

Since there are more experiments than free parameters, the model can be tested in many independent ways. Three decades and many experiments finally led to the measurement of all them, including the Higgs mass in 2012. Leaving aside the flavor-related parameters (m_f and V_{CKM}), these are listed in Table 4.

3.4 Summary of the lecture

The construction of the SM and the derivation of its fundamental properties have been the subject of this lecture, central to the course. Without any doubts, the SM constitutes one of the greatest scientific achievements of mankind. However, as we will see in the next lecture, it cannot be the final truth, as several indications clearly points towards new physics beyond the SM.

3.5 Exercises

Exercise 2.1 Consider the SM extended with a real scalar Ω with quantum numbers $(1,3)_0$ under the SM gauge group and decomposed in $SU(2)_L$ components as

$$\Omega = \begin{pmatrix} \Omega^+ \\ \Omega^0 \\ \Omega^- \end{pmatrix} . \tag{175}$$

Show that $\langle \Omega^0 \rangle \neq 0$ implies $\rho \neq 1$.

Exercise 2.2 Show that $\nu_e \overline{\nu}_e \to W_L^+ W_L^-$ has a good high-energy behavior in the SM.