4 Lecture 3: Beyond the Standard Model

4.1 Why to go beyond: experimental vs theoretical reasons

The SM is a very successful description of particle physics phenomena at energies up to the TeV scale, the energy region that our experiments have been able to explore so far. Therefore, one may naively think that we have no reason to extend the model. However, as we will learn in this lecture, this theory for the strong and electroweak interactions has several experimental and theoretical problems, thus making it necessary to go beyond the SM (BSM).

One can generally classify the reasons to go beyond the SM into two major categories:

4.2 Experimental reasons

Neutrino masses

Based on the fact that neutrinos were always observed to be left-handed, as opposed to the other fermions that could be found with both chiralities, and because no experimental result pointed to a non-zero neutrino mass, the fathers of the Standard Model decided not to add right-handed neutrinos to the particle spectrum. As we saw in the previous lecture, without right-handed neutrinos it is not possible to write down a Yukawa term that can lead to Dirac masses for neutrinos. Therefore, neutrinos are massless in the Standard Model and there is no leptonic mixing matrix.

This *practical choice* has recently been shown to be wrong: the existence of non-zero neutrino masses and mixing is nowadays an established fact thanks to neutrino oscillation experiments. Let us briefly discuss the solar and atmospheric neutrino problems, two puzzles that have finally required the introduction of neutrino masses for their resolution:

- **Solar neutrino problem:** The Sun produces neutrinos in the nuclear reactions that continuously occur in its interior. These neutrinos escape in all directions, some of them reaching the Earth and our detectors. We can now compare the predicted neutrino fluxes with the observation and check whether there is agreement between our theoretical expectation and the experimental measurements. Several experimental collaborations precisely did this, and surprisingly all of them detected less neutrinos than predicted.

- **Atmospheric neutrino problem:** Neutrinos are also produced in the atmosphere. When a cosmic ray hits an air molecule in the higher parts of the atmosphere, a particle shower is produced, including some neutrinos that travel towards the Earth, where detectors are placed underground waiting for them. As for solar neutrinos, again the predictions did not match the observations.

These puzzles are nowadays understood in terms of neutrino flavor oscillations, a phenomenom that only works if neutrinos are massive and there is a non-diagonal leptonic mixing matrix.

In fact, as soon as neutrinos are massive, the *trick* that we used in the previous lecture to eliminate the leptonic mixing matrix is not valid anymore. The U_ν matrix, the unitary transformation linking the original neutrino gauge eigenstates (also known as flavor eigenstates in this context) ν_L with the neutrino mass eigenstates $\widehat{\nu}_L$,

$$\nu_L = U_\nu \widehat{\nu}_L \, , \tag{176}$$

becomes physical. Therefore, in the presence of massive neutrinos (and regardless of the neutrino mass mechanism), the lepton charged current interaction Lagrangian becomes

$$\mathcal{L}_{\rm cc}^\ell = \frac{g}{\sqrt{2}} \overline{\widehat{\nu}}_L \gamma_\mu V_{\rm PMNS} \widehat{e}_L W^{+\mu} + \text{h.c.} \, , \tag{177}$$

where we have defined

$$V_{\rm PMNS} = U_\nu^\dagger U_e \, . \tag{178}$$

$V_{\rm PMNS}$ is a 3×3 unitary matrix, obtained from the product of the left neutrino and charged lepton rotations. This analog of the CKM matrix is the so-called Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [29].

We are now in position to discuss neutrino oscillations. This phenomenom can be easily described in just a few words: if neutrinos are massive and the flavor and mass bases are different, the neutrino flavor changes while they propagate. As a consequence, a neutrino which is originally produced as electron neutrino can be detected as muon or tau neutrino. This oscillating effect explains the deficits found in solar and atmospheric neutrino experiments. This quantum mechanical phenomenom, first discussed by Pontecorvo in 1967 [30], necessarily requires that neutrinos have non-zero masses and mixings, as one can observe by inspecting the probability for a neutrino flavor eigenstate ν_α with energy E to oscillate into a neutrino flavor eigenstate ν_β,

$$P(\nu_\alpha \to \nu_\beta) = \sum_{j,k=1}^{3} V_{\rm PMNS}^{\alpha k *} V_{\rm PMNS}^{\beta k} V_{\rm PMNS}^{\alpha j} V_{\rm PMNS}^{\beta j *} \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right) \, . \tag{179}$$

Figure 8: Masses of the known fundamental particles. For the neutrino, the conservative upper bound $m_\nu = 1$ eV is used and only one generation is represented. Leptons are drawn in blue, quarks in red and massive gauge bosons in green. The massless gauge bosons, photon and gluon, are not included in the plot. Similarly, we have not included the recently discovered Higgs boson, whose mass is about ~ 125 GeV.

Here $\Delta m^2_{kj} \equiv m^2_k - m^2_j$. Notice that the oscillation probability formula in Eq. (179) depends on squared mass differences, and not on the masses themselves. For this reason, oscillation experiments cannot tell us the absolute values of neutrino masses, for which we only have upper bounds (of the order of the eV).

A simple extension of the SM that can account for neutrino masses is the addition of 3 generations of right-handed neutrinos ν_R, with quantum numbers $(1,1)_0$ under the SM gauge group [5]. This would allow us to write new Yukawa couplings

$$\mathcal{L}_Y^\nu = Y_\nu \, \ell_L \widetilde{\Phi} \nu_R + \text{h.c.} \,, \tag{180}$$

with Y_ν a 3×3 matrix. Then, after the SSB of the electroweak symmetry one obtains Dirac neutrino masses, exactly in the same way as for the other fermions,

$$\mathcal{L}_m^\nu = \mathcal{M}_\nu \, \overline{\nu}_L \nu_R + \text{h.c.} \,. \tag{181}$$

However, this is not a popular solution in the community. The reason is that neutrinos are required by experimental data to be much lighter than the other fermions. Fig. 8 shows the masses of the known fundamental particles. For the neutrino, the conservative upper bound $m_\nu = 1$ eV is used and only one generation is represented. Note the huge difference between the upper bound for the neutrino mass and the masses of the other particles. This can be hardly understood if they share the Higgs mechanism as a common source. In fact, if we insisted on this solution, we would find that in order to obtain neutrino masses of the order of ~ 1 eV, one would require tiny Yukawas $Y_\nu \sim 10^{-11}$.

For this reason, most theorists think that a *good* neutrino mass model not only should generate neutrino masses, but it should also be able to account for their smallness. The most popular model that can do this is the famous Type-I Seesaw [31–34]. In fact, we almost found this model when we introduced right-handed neutrinos into the SM. However, before we complete the task, let us comment on the two type of fermions that can exist: Dirac and Majorana fermions.

So far, all the massive fermions that have appeared in our discussion are Dirac fermions, with mass terms of the form

$$m_D \overline{f}_L f_R + \text{h.c.} = m_D \overline{f} f \,, \tag{182}$$

see for example Eq. (181). It is easy to show that a fermion with a mass term of this type is not its own antiparticle: $f \neq f^c$. However, there is another possibility, as Majorana showed in 1937 [35]. The Lorentz symmetry also allows to write down mass terms of the form

$$\frac{1}{2} m_M \overline{f^c_X} f_X + \text{h.c.} \quad , \quad \text{with } X = L \text{ or } R \,. \tag{183}$$

In this case, and contrary to the Dirac case, one finds that a Majorana fermion is its own antiparticle: $f = f^c$. This implies that a Majorana mass term would break all $U(1)$ charges carried by the f fermion. In fact, it is clear that the term $\overline{f^c_X} f_X$ would not be invariant under any $U(1)$ transformation under which f is charged. Therefore, only fields neutral under all the conserved $U(1)$ charges of the model can be Majorana fermions [6].

[5] In fact, current neutrino data can be explained just with 2 generations of right-handed neutrinos, but is common to introduce 3 for similarity with the other fermions.

[6] For a detailed characterization of Majorana neutrinos in gauge theories see [36].

We go back to the Type-I Seesaw. In addition to the neutrino Yukawa coupling in Eq. (180), the SM gauge symmetry allows us to write down a Majorana mass term for the right-handed neutrinos. With this additional piece the part of the lagrangian that involves the right-handed neutrino becomes

$$\mathcal{L}_Y^\nu = Y_\nu \, \ell_L \widetilde{\Phi} \nu_R + \frac{1}{2} \, \overline{\nu_R^c} M_R \nu_R + \text{h.c.} \, . \tag{184}$$

Here M_R is a 3×3 symmetric matrix. As noted above, a Majorana mass breaks all $U(1)$ symmetries. Since the right-handed neutrino hypercharge is zero, the gauge symmetry is preserved. However, if we consider a lepton number symmetry, $U(1)_L$, under which all leptons are charged, the Majorana mass M_R would necessarily break it by two units. Actually, this piece not only breaks lepton number, but also changes the picture completely. After the electroweak SSB the Lagrangian (184) leads to

$$\mathcal{L}_m^\nu = m_D \overline{\nu}_L \nu_R + \frac{1}{2} \, \overline{\nu_R^c} M_R \nu_R + \text{h.c.} = \frac{1}{2} \, \overline{\chi^c} \mathcal{M}_\chi \chi + \text{h.c.} \, , \tag{185}$$

where $\chi = \begin{pmatrix} \nu_L^c & \nu_R \end{pmatrix}^T$ and

$$\mathcal{M}_\chi = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \, . \tag{186}$$

The Majorana mass M_R of the right-handed neutrinos is a free parameter of the model. Since its origin is not tied to electroweak symmetry breaking, M_R can take any value. If we assume $M_R \gg m_D$, the matrix in equation (186) can be block-diagonalized in good approximation to give

$$\widehat{\mathcal{M}}_\chi \simeq \begin{pmatrix} m_{\text{light}} & 0 \\ 0 & M_{\text{heavy}} \end{pmatrix} \tag{187}$$

with

$$m_{\text{light}} = - \, m_D^T \cdot M_R^{-1} \cdot m_D \, , \tag{188}$$

$$M_{\text{heavy}} = M_R \, . \tag{189}$$

The mass of the light neutrinos is given by $m_\nu \equiv m_{\text{light}} \sim m_D^2 / M_R$. This, usually called *the seesaw formula*, provides a natural explanation for the observed lightness of neutrinos. Let us consider the value $m_\nu \sim 1$ eV. If, for example, we take $M_R = 10^{13}$ GeV, the Dirac mass turns out to be $m_D = \frac{v}{\sqrt{2}} Y_\nu \sim 100$ GeV, which implies Yukawa couplings of order 1, $Y_\nu \sim 1$, what can be compared to the results in our discussion on Dirac neutrinos, where we showed that the same mass for the light neutrinos implies $Y_\nu \sim 10^{-11}$ in that case. Furthermore, under the same assumption, $M_R \gg m_D$, the mass eigenstates can be approximated as $\chi_{\text{light}} \simeq \nu_L$ and $\chi_{\text{heavy}} \simeq \nu_R$. This would explain why neutrinos have always been observed to be left-handed in all performed experiments: the light states are mostly left-handed.

To conclude our discussion on neutrino mass models, let us just mention that there are other variations of the seesaw mechanism and other neutrino mass models that explain the smallness of neutrino masses by completely different means.

Dark matter

The SM lacks a valid dark matter (DM) candidate. This constitutes one of the most relevant experimental indications guiding our current theoretical efforts. Since there is a specific course on DM in this school, let us just briefly review the subject for the sake of completeness.

The evidence for DM comes from many different sources. Most of it comes from the motion of galaxies and clusters. For instance, galactic rotation curves, which show the velocity of rotation of stars as a function of their distance from the galactic center, cannot be explained if all the mass is in luminous objects. This is illustrated in Fig. 9. Similar observations are made in galaxy clusters. Other indirect (but robust) evidences are obtained from gravitational lensing, the cosmic microwave background and structure formation simulations.

Even though the evidence for the existence of DM in the universe is solid, its nature is completely unknown. Among the many explanations put forward by theorists along the years, the most popular one nowadays is that DM is made of particles, just like anything else we know about. In this case, the DM particles must have some specific properties. From the model building point of view, the first three properties one must respect are:

- **Electrically neutral:** Since DM is *dark*, it should not interact with photons, at least at tree-level. Otherwise they would scatter light becoming visible.

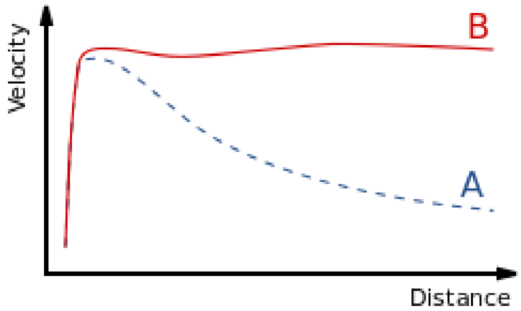


Figure 9: Rotation curve of a typical spiral galaxy: predicted (A) vs observed (B). As the distance to the galatic center increases one would expect that the velocity of luminous objects decreases (in the same way their density does). However, one observationally finds a flat dependence with the distance, suggesting the presence of additional non-luminous matter.

- **Colorless:** If DM particles were strongly interacting, like quarks, they would form bound states. This is strongly constrained by different cosmological searches.
- **Stable or long-lived:** We need the DM particles to be stable or long-lived (with a life-time of the order of the age of the universe) or otherwise they would have disappared with the evolution of the universe.

In addition, DM models must fulfill other requirements. For instance, the DM particles must be produced in the early universe in the amount required by the observed DM relic density. There are several known production mechanisms and they all involve the coupling of the DM particles to the SM states which were present in the hot plasma that filled the universe at early epochs. This typically introduces stringent constraints on the DM particle couplings to the SM states.

The only particle in the SM with the properties described above is the neutrino. However, neutrinos cannot constitute the whole of the DM of the universe but can only be a very small fraction of it. The reason is their lightness. If neutrinos have masses of the order of the eV they would constitute a hot DM component of the universe. As it is well known, this type of DM suppresses the formation of structures at small scales, of the order of $1 - 10$ Mpc, making it impossible for galaxies to form.

Therefore, if we insist on a particle explanation to the DM problem we must introduce a new particle, hence going beyond the SM. Following the requirents explained above, this particle is usually chosen to be electrically neutral, singlet under $SU(3)_c$ and absolutely stable. In principle, the possibility of a long-lived DM particle is perfectly viable and is in fact a relatively common choice in BSM models. However, from the model building point of view it is simpler to make the DM completely stable with the introduction of a symmetry.

The way a symmetry stabilizes the DM particle is quite analogous to why the proton is stable in the SM. In the SM, the gauge symmetry prevents one from writing down any renormalizable operator that breaks baryon number (B). This global symmetry is said to be *accidental* since it is not imposed when constructing the model, but just turns out to appear given our choices for the gauge symmetry and particle content. As a result of this, one can show in a straightforward way that the SM Lagrangian has a global $U(1)_B$ symmetry [7], under which all quark multiplets (q_L, u_R and d_R) transform with a baryonic charge $+1/3$, this is, as $q \to q' = \exp{(i\alpha/3)}\, q$. With this definition, the proton has baryon number (the baryonic charge) $+1$ and its stability is due to the fact that it is the lightest baryon. Since baryon number must be conserved in all decays, and given that the proton cannot decay to other (heavier) baryons due to energy conservation, it is absolutely stable.

The same mechanism can be applied to stabilize the new DM particle. One can introduce a new conserved symmetry, which might be continuous or global, local or gauge, and the lightest particle charged under this symmetry will be absolutely stable. If this particle has the desired properties for a DM particle, it is in principle a good DM candidate.

One of the simplest DM models is the so-called *singlet scalar DM* model [37]. In this case one extends the SM particle content with a real scalar $S \sim (1,1)_0$, singlet under the SM gauge group, and introduces a conserved $\mathbb{Z}_2$ symmetry, under which S is odd,

$$S \to S' = -S\,, \tag{190}$$

while all the SM particles are even (singlets under $\mathbb{Z}_2$). With these ingredients, the new Lagrangian terms are

$$\mathcal{L}_S = \frac{1}{2}\partial_\mu S \partial^\mu S - \frac{1}{2}\mu_S^2 S^2 - \frac{1}{4}\lambda_S S^4 - \frac{1}{2}\lambda_P S^2 |\Phi|^2\,. \tag{191}$$

[7] I am omitting here non-perturbative SM effects that violate $U(1)_B$. These do not affect this discussion and will be mentioned below.

Since all Lagrangian terms have even powers of S, S is completely stable. Being also electrically neutral and singlet under $SU(3)_c$, it fulfills the minimal requirements to be considered a valid DM candidate. In fact, the singlet scalar DM model has been shown to be able to accommodate the observed DM relic density. For this purpose, the λ_P coupling is crucial, as it makes the connection to the SM states (via the Higgs doublet), thus enabling the production of S particles in the early universe.

The baryon asymmetry of the universe

Finally, a third experimental reason to go beyond the SM is the baryon asymmetry of the universe.

Observations indicate that number of baryons (protons and neutrons) and antibaryons (antiprotons and antineutrons) in the universe are not equal. For example, all the structures that we see, like stars, galaxies or clusters, are made of baryons. Since various considerations suggest that the universe has started from a state with equal number of baryons and antibaryons, the observed baryon asymmetry of the universe (BAU) must be generated dynamically. This scenario is called baryogenesis.

The BAU is precisely defined as

$$Y_B \equiv \left. \frac{n_B - n_{\overline{B}}}{s} \right|_0 = (8.65 \pm 0.08) \times 10^{-11} \,, \tag{192}$$

where n_B and $n_{\overline{B}}$ are the number densities of baryons and antibaryons, respectively, s is the entropy density and the subscript 0 indicates that these quantities are measured at present time. The numerical value given here has been obtained by combining measurements from the cosmic microwave background and light element abundances (which allow us to derive a value for Y_B due to its crucial role in big bang nucleosynthesis), and it is the number our dynamical mechanism must be able to explain.

The three ingredients required to dynamically generate a BAU were given by Sakharov in 1967 and are known as *Sakharov's conditions* [38]:

1. **B violation:** baryon number must be violated in order to evolve from a state with $Y_B = 0$ to a current universe with $Y_B \neq 0$.

2. **C and CP violation:** If either C or CP were conserved, processes involving baryons would proceed at the same rate as those involving antibaryons, thus compensating each other and leading to a vanishing overall effect.

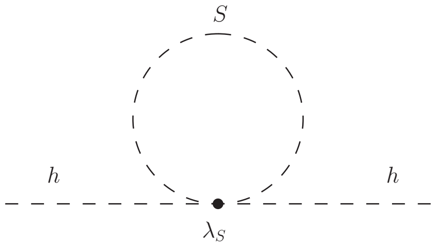
3. **Departure from thermal equilibrium:** In thermal equilibrium it is not possible to generate an asymmetry since direct ($A \to B$) and inverse ($A \leftarrow B$) processes would take place at the same rate.

These ingredients are all present in the SM but not in the right amount. B is violated in the SM by QCD triangle anomaly processes. At zero temperature they are very suppressed and no observable effects can be measured. However, at high temperatures these transitions can be effective thanks to special field configurations. These are the so-called *sphalerons*. Regarding C and CP, they are both violated by the weak interactions, as we learnt in the previous lecture. In particular, CP is violated in the quark sector due to the existence of 3 generations, which introduce a CP violating phase in the CKM matrix. However, when one quantifies the amount of CP violation it is easy to show that this is small, not enough for baryogenesis to be successful and generate the observed Y_B. And finally, departure from equilibrium is achieved when the universe cools down at temperatures around the Fermi scale. At this state the electroweak phase transition takes places. However, this is again found not to be enough. A Higgs boson with a mass of about ~ 125 GeV implies that the phase transition is not of first order, as required for electroweak baryogenesis.

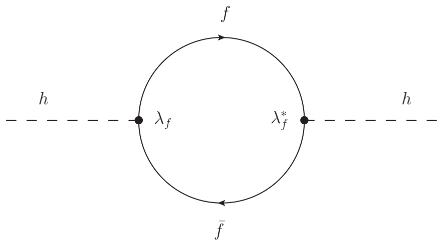
One then concludes that BSM physics is required in order to explain the BAU. Many mechanisms have been put forward along the years. Here we will just mention one, due to its connection to neutrino masses and the seesaw mechanism. This is leptogenesis.

Leptogenesis was first proposed by Fukugita and Yanagida [39]. Although the idea can be applied in many neutrino mass models with lepton number violation, its classical realization is based on the type-I seesaw. The Y_ν Yukawa couplings of the singlet right-handed neutrinos are general complex matrices and thus can provide the necessary additional source of CP violation. Furthermore, the heavy right-handed neutrinos will decay out of the thermal equilibrium when the decay rate is slower than the expansion rate of the universe, a moment that is known as their decoupling. Finally, the Majorana mass terms for the right-handed neutrinos violate L, and thus the dynamics of these decays will generate a lepton asymmetry which can be later converted into a baryon asymmetry by the sphaleron processes mentioned above, which violate $B + L$ but preserve $B - L$. As a result of this, a net baryon asymmetry can be generated.

It is clearly beyond the scope of these lectures to give a quantitative analysis of the baryon asymmetry that one can achieve in leptogenesis. However, it is instructive to discuss the main elements that play a role. For



(a) Scalar contribution



(b) Fermion contribution

Figure 10: Feynman diagrams leading to 1-loop corrections to the Higgs boson mass.

this purpose, let us consider the simplified case of a lepton asymmetry generated by the lightest heavy neutrino decay, N_1. The baryon asymmetry Y_B can be approximated as

$$Y_B = c \sum_\alpha \epsilon_{\alpha\alpha}\, \eta_\alpha\, C_\alpha\,. \tag{193}$$

Here c is a numerical factor, $\alpha = e, \mu, \tau$, $\epsilon_{\alpha\alpha}$ is the CP asymmetry in the N_1 decay, defined as

$$\epsilon_{\alpha\alpha} = \frac{\Gamma\left(N_1 \to \Phi \ell_\alpha\right) - \Gamma\left(N_1 \to \Phi^\dagger \bar{\ell}_\alpha\right)}{\Gamma\left(N_1 \to \Phi \ell\right) + \Gamma\left(N_1 \to \Phi^\dagger \bar{\ell}\right)}\,. \tag{194}$$

η_α is the efficiency factor, a generic way to parameterize the effect of the processes taking place in the thermal bath that tend to reduce the BAU. A simple example is given by the inverse decays, $\Phi \ell_\alpha \to N_1$, but in general there may be many other relevant processes. These are known as *washout processes*. Finally, C_α describes further reduction of the lepton asymmetry due to fast processes (in thermal equilibrium) which redistribute the asymmetry that is produced in the ℓ_α lepton doublets among other particle species. One can see from expression (193) that each Sakharov condition introduces a suppression factor. Therefore, one may end up with a tiny BAU unless all factors are sizable. In practice, this implies constraints on the parameter space of the model.

Before concluding our discussion on leptogenesis let us emphasize its most attractive feature: it provides a mechanism for the dynamical generation of the BAU that is directly connected to the smallness of neutrino masses via the seesaw mechanism. This makes leptogenesis a popular subject in current particle physics. For a more detailed review we recommend [40].

4.3 Theoretical reasons

We will now discuss a completely different of reasons to go beyond the SM: theoretical indications and suggestive ideas for possible extensions. These, being mor speculative, are less robust than the experimental ones discussed in Sec. 4.2. Nevertheless, they constitute equally interesting research directions in current particle physics.

The hierarchy problem

The hierarchy problem has been one of the driving forces behind the theoretical developments in the BSM community for the last decades. As we will see in the next lines, this is not a problem of the SM *per se*, but a problem that appears when the SM is supplemented with new physics at energies much higher than the Fermi scale. This is usually forgotten, leading to a general confusion regarding the hierarchy problem. Therefore, let us insist once more: if the SM was all physics that exist, there would be no hierarchy problem at all. Only when we think of the SM as the low-energy limit of a more complete theory including heavier degrees of freedom one finds the naturalness issue known as hierarchy problem.

The mass of the Higgs boson has been discussed in lecture 2, finding the tree-level result $m_h = \sqrt{-2\mu^2}$. To this, one has to add radiative corrections coming from the interactions of the Higgs boson with the rest of particles in the theory. Let us first consider a scalar S with mass m_S that couples to the Higgs boson with an interaction term of the form $-\lambda_S |h|^2 |S|^2$. Then the Feynman diagram in figure 10(a) gives a contribution

$$\left(\Delta m_h^2\right)_S \sim \lambda_S \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - m_S^2}\,. \tag{195}$$

By dimensional analysis, this contribution is proportional to m_S^2. If the scalar S is a heavy particle, with a mass much above the electroweak scale, such a quadratic correction will be much larger than the Higgs boson tree-level mass. Let us now consider a Dirac fermion f with mass m_f that couples to the Higgs boson with a

Yukawa interaction term $-\lambda_f h \bar{f} P_L f + \text{h.c.}$, where P_L is the left chirality projector. Then, its contribution to the Higgs boson mass is given by diagram 10(b) and turns out to be

$$\left(\Delta m_h^2\right)_f \propto -|\lambda_f|^2 \int \frac{d^4p}{(2\pi)^4} \frac{\text{TR}}{(p^2 - m_f^2)^2} , \tag{196}$$

where $\text{TR} = \text{Tr}[(\not{p} + m_f)P_L(\not{p} + m_f)P_R] = 2p^2$. Using this result for the fermionic trace, Eq. (196) splits into

$$\left(\Delta m_h^2\right)_f \propto -|\lambda_f|^2 \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - m_f^2} + \log , \tag{197}$$

where log corresponds to a logarithmic integral that can be absorbed by choosing the right renormalization scale. Again, there is a quadratic correction to the Higgs boson mass that is proportional to m_f^2. Analogously to the case of the scalar diagram, if the fermion f has a large mass $m_f \gg m_h$ the correction $\left(\Delta m_h^2\right)_f$ will be also much larger than the tree-level mass.

The problem appears when we think of the SM as an effective theory obtained at low energies from an extended model that describes physics at higher energies. For example, if the SM is to be embedded in a Grand Unification Theory (GUT), the corrections to the Higgs boson mass given by the particles that live at the GUT scale are, according to equations (195) and (196), proportional to the square of their masses. Since the GUT scale is expected to be at around $m_{\text{GUT}} = 10^{16}$ GeV (see below), we would have the following 1-loop prediction for the Higgs boson mass

$$\begin{aligned}\left(m_h^2\right)_{\text{1-loop}} =& - 2\mu^2 + \left(\Delta m_h^2\right)_S + \left(\Delta m_h^2\right)_f \\ & \sim (100\,\text{GeV})^2 + (10^{16}\,\text{GeV})^2 + (10^{16}\,\text{GeV})^2 \sim (100\,\text{GeV})^2 .\end{aligned} \tag{198}$$

With such large quadratic corrections it is hard to understand how the mass of the Higgs boson could be at the electroweak scale unless a very precise conspiracy among the different contributions from the heavy particles makes them cancel. This is the famous hierarchy problem.

There are several solutions to the hierarchy problem but, as we did in the previous sections, we will concentrate on just one: supersymmetry. It is simple to see how the idea may arise. If we look at Eqs. (195) and (196), we will immediately note that if

$$m_S^2 = m_f^2 \tag{199}$$

and

$$\lambda_S = |\lambda_f|^2 , \tag{200}$$

the scalar and fermion contributions cancel exactly. For that to happen there must be a reason, a symmetry that relates fermions and bosons. That symmetry is supersymmetry (SUSY).

The vast SUSY literature makes it hard to go though the main concepts in a limited amount of space. Let us mainly discuss the basic properties of SUSY models:

- SUSY is a symmetry that relates bosons and fermions. In fact, SUSY implies that for every particle in the spectrum one must add another one with the same mass but different spin. For instance, the electron must have a scalar partner (or "superpartner"), the selectron ($\widetilde{e}$). This leads to a duplication of the particles when going from a non-SUSY model to a SUSY one.

- However, this poses a problem, since a charged scalar with the mass of the electron would have been already discovered. This is because, if SUSY is realized in nature, it cannot be an exact symmetry. It must be broken. This way, particles in the same supermultiplet would have different masses as needed to account for the non-discovery of the superpartners of the SM fermions. Unfortunately, the way SUSY is broken is unknown. In practice, this ignorance is solved by introducing by hand new terms in the Lagrangian that break SUSY explicitly but preserve the solution that SUSY offers for the hierarchy problem.

- Supersymmetric models are typically supplemented with a discrete symmetry called R-parity. All superparticles (like the selectron) have R-parity -1, while the SM particles have R-parity $+1$. This parity, introduced in order to forbid some dangerous L and B violating interactions, also serves to stabilize the lightest supersymmetric particle (LSP), which in this way can be used as a DM candidate provided it is neutral and colorless. Furthermore, the conservation of R-parity also leads to characteristic signatures at colliders, with the requirement that all superparticles must be produced in pairs and with the presence of large amounts of missing energy in all SUSY events.

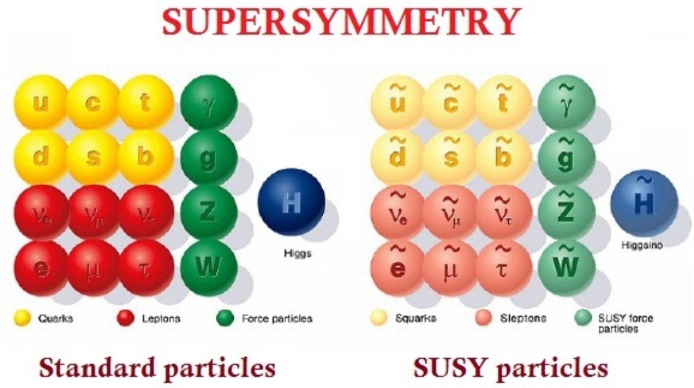


Figure 11: MSSM particle content. We note that the usual SM Higgs doublet Φ is actually duplicated in the MSSM, with the existence of the doublets H_u and H_d (and the corresponding superpartners $\widetilde{H}_u$ and $\widetilde{H}_d$).

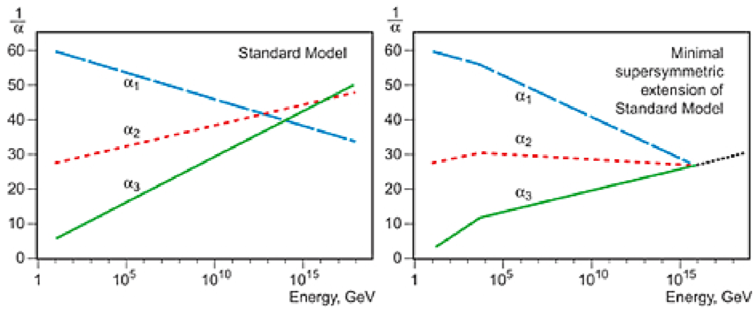


Figure 12: Running of the gauge couplings in the SM (left) and the MSSM (right). The curves show the evolution of α_i^{-1}, where $\alpha_i = \frac{g_i^2}{4\pi}$, $g_1 = \sqrt{5/3}\, g'$, $g_2 = g$ and $g_3 = g_s$. This figure includes contributions to the running up to the 1-loop level.

The simplest (and realistic) supersymmetric model is the Minimal Supersymmetric Standard Model (MSSM) [41], whose particle content is represented in Fig. 11. Many extensions of this model exist, with additional "superfields" (as we call SUSY multiplets containing a particle and its superpartner) and/or symmetries [8].

Although the theoretical motivation for SUSY is strong, the experimental results do not favor it. Many SUSY searches have been performed at the Large Hadron Collider (LHC), finding no hint of it. This has been used to establish stringent bounds on the masses of the superparticles, which in some cases are pushed clearly above the TeV scale. This is of course fine in what concerns the consistency of the theory, as no upper bound on the superparticles masses can be derived from first principles. However, it weakens the motivation for SUSY, since a large mass splitting between the SM particles and their superpartners reintroduces a certain amount of hierarchy problem.

Unification

One of the most interesting predictions of QFT is the dependence of the interaction strength with the energy. Some interactions become weaker at high energies while others become stronger. A famous example of this phenomenom is *asymptotic freedom*, a property of QCD discovered by Gross, Wilczek and Politzer [42, 43] in 1973, which implies that the strong interactions become asymptotically weaker as energy increases.

The question is: what happens to the three SM gauge couplings, g, g' and g_s, at high energies? Do they approach a common value or they split never to meet again? This is illustrated in Fig. 12. The running depends on the particle content of the model, and therefore it is different in the SM and the MSSM. In both cases they approach a common region at high energies. However, while in the SM case they do not match at a single point, in the MSSM the coincidence at energies around $\sim 10^{16}$ GeV is quite good. This suggests an interesting possibility: the three gauge groups of the SM, $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$, may be unified in a single group G_{GUT} above the unification scale. As we see, this possibility is more favorable in the MSSM than in the SM. Then, after SSB of G_{GUT}, the unified force is not unified anymore, and the three symmetry groups of the SM become independent. At low energies they appear to be very different due to the large energy gap from the GUT scale down to the Fermi scale, which leads to sizable individual runnings. This is Grand Unification.

[8] In fact, neutrinos are massless in the MSSM, in the same way they are in the SM. Therefore, neutrino masses and mixings call for an extension of the MSSM, and the known non-SUSY solutions can be applied in the supersymmetric case as well.

There are many Grand Unified Theories (GUTs), with different symmetry groups and fields, but they all share some common properties:

- The SM gauge group is a subgroup of $G_{\rm GUT}$.
- The SM particles are embedded into larger multiplets, with definite transformation properties under $G_{\rm GUT}$.
- In some cases, the embedding requires the addition of new particles, which allow to complete the multiplets, while in other cases the SM multiplets are combined (grouped) to form larger multiplets.
- The breaking of the GUT symmetry is realized in the same way as in the SM: by the Higgs mechanism. This implies the introduction of large scalar multiplets with VEVs of the order of the new energy scale $m_{\rm GUT} \sim 10^{16}$.

The most popular GUTs are those based on the $SU(5)$ [44] and $SO(10)$ [45,46] groups. These are complicated theoretical constructions and we will not review them here. However, let us just mention one attractive feature of GUT models: charge quantization.

The fact that $q_e + q_p = 0$, with q_e the electric charge of the electron and q_p the electric charge of the proton, does not have an explanation within the SM. The electric charges of the quarks and leptons could be different, not necessarily integer multiples of 1/3, and this equality would not hold. Moreover, this relation is more surprising given that they are in different SM multiplets. What about in a unified model in which they are embedded in the same multiplet? In fact, in unified models the quantum numbers of quarks and leptons are related by the gauge symmetry, in the same way the neutrino and electron (or up and down quarks) quantum numbers are related in the SM. This connection automatically leads to the quantization of charge or, in other words, to $q_e + q_p = 0$. This elegant *postdiction* of GUT models is one of their most appealing properties.

The most clear experimental prediction of GUTs is proton decay. Since leptons and quarks are embedded in the same GUT multiplets, the GUT gauge bosons mediate L and B violating interactions. Therefore, processes like $p \to e^+ \pi^0$ should be possible. However, and despite the experimental efforts in the search for proton decay, this has never been observed.

The flavor problem

The fermionic content of the SM consists of 3 copies of the same set of states. These 3 generations or families have exactly the same gauge quantum numbers and only differ by their Yukawa couplings to the Higgs doublet. This raises three fundamental questions:

- Why are there 3 fermionic replicas?
- What is the origin of the quark and lepton masses?
- What is the origin of the observed patterns of the Yukawa couplings?

The first question is due to the fact that the SM would be perfectly consistent with only one fermion family. Therefore, there is no clue in the SM itself about the reason for 3 generations, and not any other specific number. Figure 8 is at the origin of the second question: why the top quark is about 6 orders of magnitude heavier than the electron? Finally, the motivation for the third question can be visualized more easily by looking at the measured structure of the CKM and PMNS matrices. Numerically, the absolute values of the CKM matrix elements have been measured to be [47] (we only give central values)

$$|V_{\rm CKM}| = \begin{pmatrix} 0.974254 & 0.22542 & 0.003714 \\ 0.22529 & 0.973394 & 0.04180 \\ 0.008676 & 0.04107 & 0.999118 \end{pmatrix} . \tag{201}$$

We see that the CKM matrix is almost diagonal, implying that the angles defined in Eq. (166) are small. On the other hand, the PMNS matrix does not have any clear structure, with the three mixing angles being large [48] (again, we only give central values)

$$|V_{\rm PMNS}| = \begin{pmatrix} 0.813449 & 0.561872 & 0.150333 \\ 0.467118 & 0.47709 & 0.744437 \\ 0.346556 & 0.675785 & 0.650549 \end{pmatrix} . \tag{202}$$

These open questions, which the SM cannot address, are usually referred to as the flavor problem.

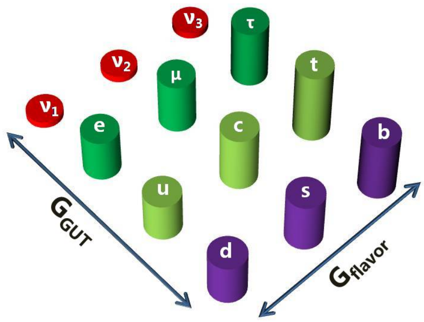


Figure 13: An illustrative diagram showing the action of flavor ("horizontal") symmetries and gauge ("vertical") symmetries, such as those in GUT models.

Again, there are many ideas to address these issues. The most popular ones involve "flavor symmetries", symmetries that instead of acting *vertically* on different types of fermions, act *horizontaly* by grouping together fermions of the same type within different families. For example, one can embed the 3 fermion families in $SU(3)_f$ triplets. By properly breaking the symmetry one can aim at inducing the observed fermion mixing patterns and, more ambitiously, to explain the structure and hierarchies of the Yukawa couplings.

4.4 Summary of the lecture

In this last lecture we have discussed several SM problems and some popular solutions put forward to address them. First, we concentrated on experimental problems, this is, problems that have been revealed by experimental measurements. These simply cannot be ignored. And second, we discussed several theoretical problems and indications, less robust but equally interesting in our search for a new physics paradigm beyond the SM.

4.5 Exercises

Exercise 3.1 The Type-II Seesaw. Consider an extension of the SM by a scalar Δ with quantum numbers $(1,3)_1$ under the SM gauge group and decomposed in $SU(2)_L$ components as

$$\Delta = \begin{pmatrix} \Delta^{++} \\ \Delta^{+} \\ \Delta^{0} \end{pmatrix} . \tag{203}$$

Write the most general Lagrangian allowed by the SM gauge symmetry and show that a non-zero $\langle \Delta^0 \rangle$ induces Majorana masses for the left-handed neutrinos.

Exercise 3.2 Consider the running of the gauge couplings in the SM and in the MSSM and reproduce the results of Fig. 12. For this purpose use the renormalization group equation

$$\frac{d}{dt}\alpha_i^{-1} = -\frac{b_i}{2\pi} \, . \tag{204}$$

Here $t = \log Q$ and Q is the renormalization scale. The b_i coefficients are given by

$$(b_1, b_2, b_3) = \begin{cases} (41/10, -19/6, -7) & \text{SM} \\ (33/5, 1, -3) & \text{MSSM} \end{cases} , \tag{205}$$

and we have defined $\alpha_i = \frac{g_i^2}{4\pi}$, $g_1 = \sqrt{5/3}\, g'$, $g_2 = g$ and $g_3 = g_s$.